Math 55 Quiz 9 DIS 105

Name: ______________________________ 25 Apr 2022

1. A relation R is called circular if aRb and bRc imply that cRa. Show that R is reflexive and circular if and only if it is an equivalence relation. [4 points]

   Suppose R is reflexive and circular. To show that R is an equivalence relation, we need to check that it is reflexive, symmetric, and transitive.
   R is reflexive by assumption.
   To show that R is symmetric: Suppose $(x, y) \in R$, then $(x, y) \in R$ and $(y, y) \in R$ by reflexiveness, so $(y, x) \in R$ by circularity.
   To show that R is transitive: Suppose $(x, y) \in R$ and $(y, z) \in R$, then $(z, x) \in R$ by circularity, so $(x, z) \in R$ by symmetry proven above.
   Conversely, suppose R is an equivalence relation. Then R is reflexive by definition.
   To show that R is circular: Suppose $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$ by transitivity hence $(z, x) \in R$ by symmetry.

2. Give an example of a relation on the set $\{1, 2, 3, 4\}$ that is

   (a) reflexive, symmetric, and not transitive. [2 points]

   (b) not reflexive, symmetric, and transitive. [2 points]

   (c) reflexive, antisymmetric, and transitive. [2 points]

   There are many possible answers here, we simply write down one example for each question.

   (a) $\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 3), (2, 1), (3, 1)\}$

   (b) $\{(1, 2), (2, 1)\}$

   (c) $\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\} = \{(a, b)|a \leq b\}$